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De Casteljau Algo

## Bezier Curve:

$$p(x) = \sum_{i=0}^{n} B_i C_i \quad , \quad B_i = \binom{n}{i} x^i (1-x)^{n-i}$$

$$C_j^i = (1-x) C_j^{i-1} + x C_{j+1}^{i-1} \quad , \quad i = 0, \dots, n \; , \; j = 0, \dots, n-i$$

$$C_i^0 = C_i$$

$$p(x) = \sum_{i=0}^{n+1} B_i C_i$$

$$C_0^{n+1} = (1-x) C_0^n + x C_1^n$$

$$= (1-x) \sum_{i=0}^{n} B_i C_i + x \sum_{i=1}^{n+1} B_i C_i$$

compare coefficient of $C_i$:

$$(1-x) \binom{n}{i} x^i (1-x)^{n-i} + x \binom{n}{i-1} x^{i-1} (1-x)^{n-i+1}$$

$$= \left[ \binom{n}{i} + \binom{n}{i-1} \right] x^i (1-x)^{n-i+1}$$

$$= \binom{n+1}{i} x^i (1-x)^{n+1-i}$$

//

$$\binom{n}{i} + \binom{n}{i-1} = \frac{n!}{i!(n-i)!} + \frac{n!}{(i-1)!(n+1-i)!}$$

$$= \frac{n!}{(i-1)!(n-i)!} \left( \frac{1}{i} + \frac{1}{n+1-i} \right)$$

$$= \frac{(n+1)!}{i!(n+1-i)!} = \binom{n+1}{i}$$

Jump Conditions

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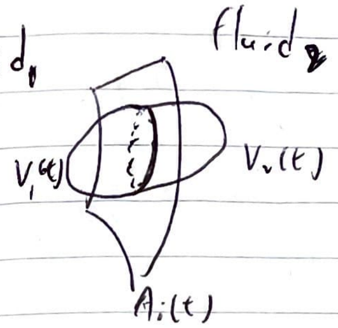


$v_1^t = v_2^t$

$T_1 \neq T_2$

$v_i$ = interface velocity

$v_p$ = flow velocity at interface $= (v_i \cdot \hat{n})\hat{n} + v^t$

mass balance:

$$\boxed{\frac{d}{dt}\int_{V_1(t)} \rho_1 \, dV + \frac{d}{dt}\int_{V_2(t)} \rho_2 \, dV + \frac{d}{dt}\int_{A_i(t)} \rho_i \, dA = 0}$$

assume $\rho_1, \rho_2$ are constant

by Reynold's transport theorem,

surface divergence

$$\int_{V_1(t)} \frac{d\rho_1}{dt} dV + \int_{V_2(t)} \frac{d\rho_2}{dt} dV + \int_{A_1(t)} \rho_1 (v_1 - v_i)\cdot \hat{n}_1 \, dA + \int_{A_2(t)} \rho_2 (v_2 - v_i)\cdot \hat{n}_2 \, dA = \int_{A_i(t)} \left(\frac{d\rho_i}{dt} + \rho_i \nabla_s \cdot v_p\right) dA$$

let $V_1, V_2$ ~~$A_1, A_2, A_3$~~ $\to 0$

$$\therefore \boxed{\rho_1 (v_1 - v_i)\cdot \hat{n}_1 + \rho_2 (v_2 - v_i)\cdot \hat{n}_2 = \frac{d\rho_i}{dt} + \rho_i \nabla_s \cdot v_p}$$

momentum balance:

$$\frac{d}{dt}\int_{V_1(t)} \rho_1 v_1 \, dV + \frac{d}{dt}\int_{V_2(t)} \rho_2 v_2 \, dV + \frac{d}{dt}\int_{A_i(t)} \rho_i v_p \, dA$$

linear momentum

$$= \int_{V_1(t)} \rho_1 F_1 \, dV + \int_{V_2(t)} \rho_2 F_2 \, dV + \int_{A_i(t)} \rho_i F_i \, dA$$

external forces

$$+ \int_{A_1(t)} \hat{n}_1 \cdot \Pi_1 \, dA + \int_{A_2(t)} \hat{n}_2 \cdot \Pi_2 \, dA + \oint_{l(t)} \sigma N \, dl$$

viscous force & surface tension

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$$\int_{V_1(t)} \frac{d}{dt}(\rho_1 v_1)\, dV + \int_{A_1(t)} \rho_1 v_1 (v_1 - v_i)\cdot \hat{n}\, dA + \int_{V_2(t)} \frac{d}{dt}(\rho_2 v_2)\, dV$$

$$+ \int_{A_2(t)} \rho_2 v_2 (v_2 - v_i)\cdot \hat{n}\, dA + \int_{A_i(t)} \left[\frac{d}{dt}(\rho_i v_p) + \rho_i v_p\, \nabla_s \cdot v_p\right] dA$$

$$= \int_{V_1(t)} \rho_1 F_1\, dV + \int_{V_2(t)} \rho_2 F_2\, dV + \int_{A_i(t)} \rho_i F_i\, dA + \int_{A_1(t)} \hat{n}_1 \cdot \Pi_1\, dA + \int_{A_2(t)} \hat{n}_2 \cdot \Pi_2\, dA$$

$$+ \oint_{l(t)} \sigma N\, dl$$

let $V_1, V_2 \to 0$

$$\therefore \boxed{\begin{aligned} &\rho_1 v_1 (v_1 - v_i)\cdot \hat{n}_1 + \rho_2 v_2 (v_2 - v_i)\cdot \hat{n}_2 - \hat{n}_1 \cdot \Pi_1 - \hat{n}_2 \cdot \Pi_2 \\ &= \rho_i \frac{dv_p}{dt} + v_p\left(\frac{d\rho_i}{dt} + \rho_i \nabla_s \cdot v_p\right) + \nabla_s \sigma - (\nabla_s \cdot \hat{n})\sigma \hat{n} \end{aligned}}$$

$$\oint \sigma N\, dl = \oint \sigma\, ds \times \hat{n}$$

$$= \oint ds \times \hat{n}\sigma$$

$$= \int_A \hat{n} \times \nabla \times (\hat{n}\sigma)\, dA$$

$$= \int_A \left[(\hat{n} \times \nabla \times \hat{n})\cdot \sigma + (\hat{n} \times \nabla \sigma) \times \hat{n}\right] dA$$


let $\nabla = \nabla_n + \nabla_s$, $\therefore$ ~~$\hat{n} \times \nabla \times \hat{n}$~~ $\hat{n} \times \nabla = \hat{n} \times (\nabla_n + \nabla_s) = \hat{n} \times \nabla_s$

normal ↑ tangential (on surface)

$$\hat{n} \times \nabla_s \times \hat{n} = \varepsilon_{ijk}\, \hat{n}_j\, \nabla_{s_k}\, \varepsilon_{min}\, \hat{n}_n$$

$$= (\delta_{jn}\delta_{km} - \delta_{jm}\delta_{nk})\, \hat{n}_j\, \nabla_{s_k}\, \hat{n}_n$$

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$$= \hat{n}_j D_{s_m} \hat{n}_j - \hat{n}_m D_{s_k} \hat{n}_k$$

$$= \frac{1}{2} D_s (\hat{n} \cdot \hat{n}) - \hat{n} D_s \cdot \hat{n}$$

$$= - \hat{n} D_s \cdot \hat{n} \quad //$$

similarly, $(\hat{n} \times D\sigma) \times \hat{n} = \hat{n}_j D_{s_m} \sigma \hat{n}_j - \hat{n}_m \hat{n}_k D_{s_k} \sigma$

$$= D\sigma - \hat{n}(\hat{n} \cdot D)\sigma = D_s \sigma$$

$$\therefore \oint \sigma N \, dl = \int_A (D_s \sigma - \sigma \hat{n} D_s \cdot \hat{n}) \, dA$$

Energy balance:

$$\boxed{\begin{aligned} &\frac{d}{dt} \int_{V_1(t)} \rho_1 \left(\frac{1}{2} v_1^2 + u_1\right) dV + \frac{d}{dt} \int_{V_2(t)} \rho_2 \left(\frac{1}{2} v_2^2 + u_2\right) dV + \frac{d}{dt} \int_{A_i(t)} \rho_i \left(\frac{1}{2} v_p^2 + u_i\right) dA \\ &= \int_{V_1(t)} \rho_1 F_1 \cdot v_1 \, dV + \int_{V_2(t)} \rho_2 F_2 \cdot v_2 \, dV + \int_{A_i(t)} \rho_i F_i \cdot v_p \, dA \\ &\quad + \int_{A_1(t)} (\hat{n}_1 \cdot \Pi_1) \cdot v_1 \, dA + \int_{A_2(t)} (\hat{n}_2 \cdot \Pi_2) \cdot v_2 \, dA + \oint_{l(t)} \sigma v_p \cdot N \, dl \\ &\quad - \int_{A(t)} q_1 \cdot \hat{n}_1 \, dA - \int_{A(t)} q_2 \cdot \hat{n}_2 \, dA - \oint_{l(t)} q_i \cdot N \, dl \end{aligned}}$$

similar to momentum balance, we get

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$$\boxed{\begin{aligned}
&\rho_1\left(\tfrac{1}{2}v_1^2 + u_1\right)(v_1 - v_i)\cdot\hat{n}_1 + \rho_2\left(\tfrac{1}{2}v_2^2 + u_2\right)(v_2 - v_i)\cdot\hat{n}_2 + q_1\cdot\hat{n}_1 + q_2\cdot\hat{n}_2 \\
&\quad = \cancel{\frac{d}{dt}\left(\tfrac{1}{2}v_p^2 + u_i\right)\frac{d\rho_i}{dt} + \rho_i\left(\tfrac{1}{2}\right.} - (\hat{n}_1\cdot\pi_1)\cdot v_1 - (\hat{n}_2\cdot\pi_2)\cdot v_2 \\
&\quad = \rho_i\frac{d}{dt}\left(\tfrac{1}{2}v_p^2 + u_i\right) + \left(\tfrac{1}{2}v_p^2 + u_i\right)\left(\frac{d\rho_i}{dt} + \rho_i D_s\cdot v_p\right) + \rho_i F_i\cdot v_p \\
&\qquad + D_s q_i + D_s\cdot(\sigma v_t)
\end{aligned}}$$